Ground-state energy distribution of noninteracting fermions with a random energy spectrum

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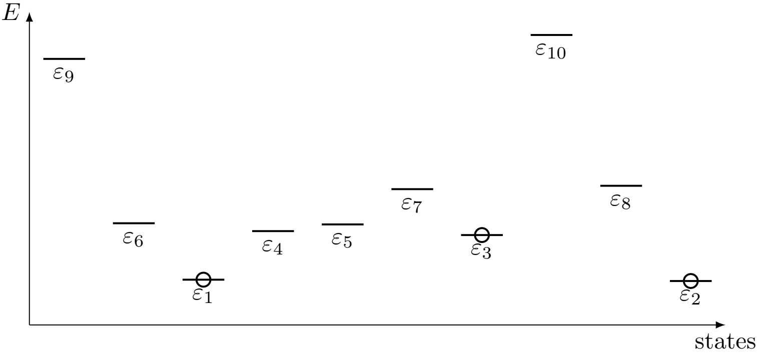
Outline

Ground state energy distribution of a random energy model

- N energy levels ε_i, $(\varepsilon_1 \leq \varepsilon_2 \leq .. \leq \varepsilon_N)$
- independent, identical distributed $p(\varepsilon), \varepsilon \geq 0$
- K levels are occupied
- ground state energy $E_0 = \sum_{i=1}^{K} \varepsilon_i$
- inspired by a generalized spin glass model Derrida (1980)



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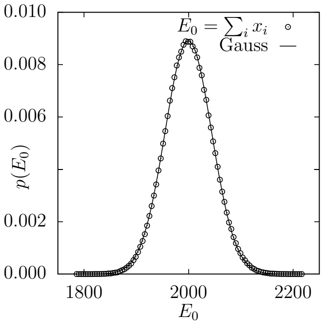
Extrema

$K = N$

central limit theorem

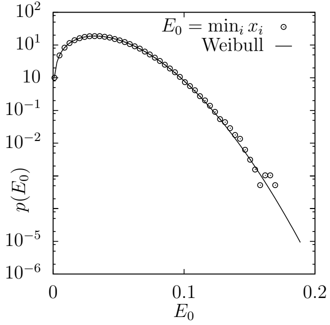
Gaussian distributed



$K = 1$

extreme value theory

Weibull distributed



What about intermediate K?

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Results of G. Schehr and S. Majumdar

Starting from

$$P(\varepsilon_1, \cdots, \varepsilon_K) = \frac{\Gamma(N+1)}{\Gamma(N-K+1)} \prod_{i=1}^{K} p(\varepsilon_i) \prod_{i=2}^{K} \Theta(\varepsilon_i - \varepsilon_{i-1}) \left[\int_{\varepsilon_K}^{\infty} p(u)\ \mathrm{d}u\right]^{N-K}$$

the scaling form for $N \to \infty$ is obtained

$$P_{K,N}(E_0) \approx b\, N^{\frac{1}{\alpha+1}} F_K^{(\alpha)} \left(b\, N^{\frac{1}{\alpha+1}}\, E_0\right)$$

with an expression for $F_K^{(\alpha)}$

$$\int_0^{\infty} F_K^{(\alpha)}(z) e^{-\lambda\, z}\ \mathrm{d}z = \frac{(\alpha+1)^K}{\Gamma(K)\lambda^{(\alpha+1)(K-1)}} \int_0^{\infty} x^{\alpha} e^{-\lambda\, x - x^{\alpha+1}} \left[\gamma(\alpha+1, \lambda\, x)\right]^{K-1}\ \mathrm{d}x$$

universal with two parameters

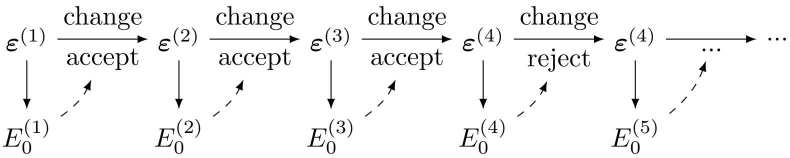
$$p(\varepsilon) \overset{\varepsilon\to 0}{\approx} B\varepsilon^{\alpha}$$

$$b = (B/(\alpha+1))^{1/(\alpha+1)}$$

Metropolis Algorithm

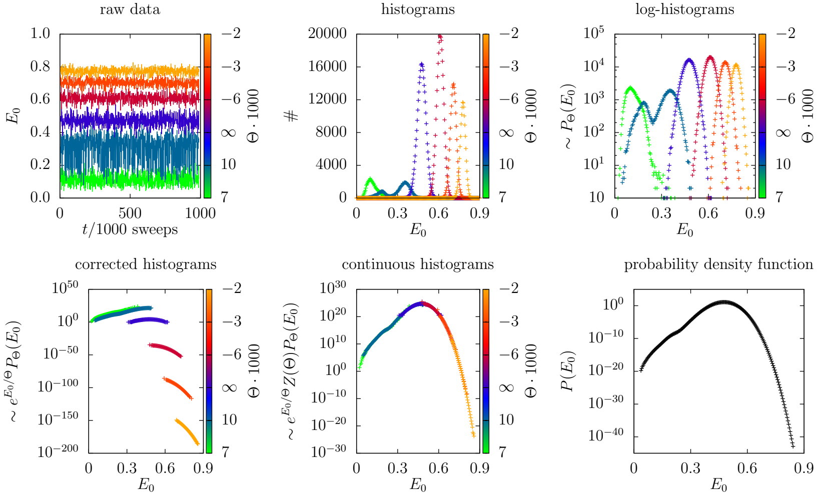
- treat it as a canonical ensemble, i.e., weights $\sim e^{-E_0/\Theta}$
- artificial temperature Θ for the disorder ($\boldsymbol{\varepsilon}$)
- Markov chain of realizations $\boldsymbol{\varepsilon} = (\varepsilon_1, .., \varepsilon_N)$
- accept change with probability

$$p_{\text{acc}} = \min\left\{1, e^{-\Delta E_0/\Theta}\right\}$$



Metropolis et al., 1954

Large Deviation Simulation Sketch



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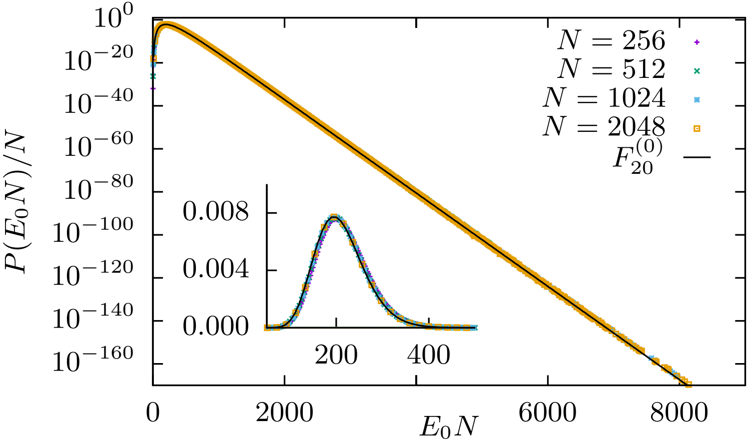
Numerical Results

exponentially distributed ε ($\alpha = 0, B = 1$), $K = 20$

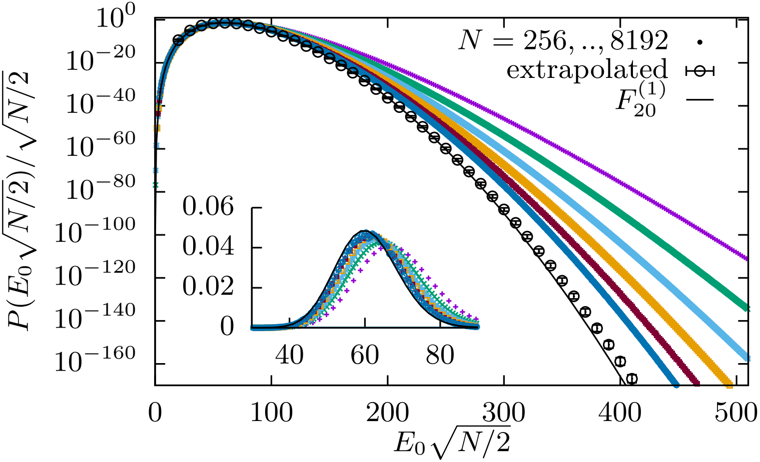
$$p(\varepsilon) = e^{-\varepsilon}, \varepsilon > 0$$



Numerical Results

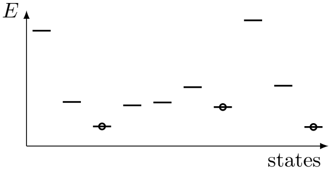
Erlang-distributed ε ($\alpha = 1, B = 1$), $K = 20$

$$p(\varepsilon) = \varepsilon e^{-\varepsilon}, \varepsilon > 0$$

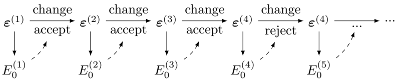


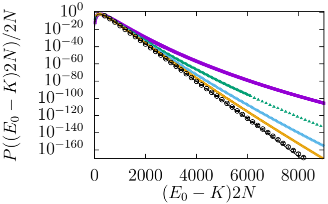
Summary

- simple random-energy model
- asymptotics of ground-state energy distribution by analytic calculations
- MCMC simulations confirm the asymptotics over vast parts of the distribution



$$P_{K,N}(E_0) \approx b\, N^{\frac{1}{\alpha+1}}\, F_K^{(\alpha)}\left(b\, N^{\frac{1}{\alpha+1}}\, E_0\right)$$





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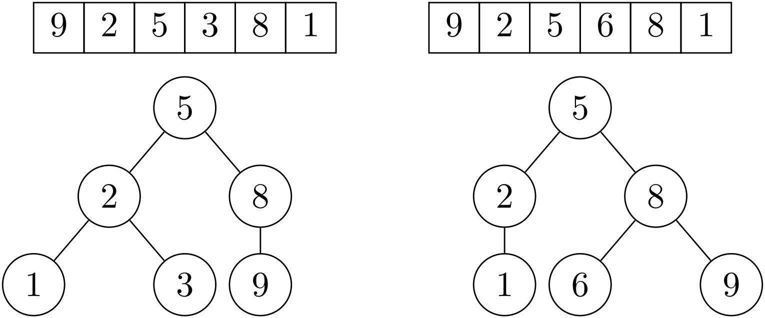
Efficient data structure for change move



Change move is problematic!

- for very large values E_0 every single ε_i must be atypically large
- replacing one with a new uniform ε_i will in most cases not increase E_0

Solution:

- operate on uniform random numbers $\boldsymbol{\xi}$ with $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}(\boldsymbol{\xi})$
- change every entry only slightly $\xi_i \to \xi_i + 10^{-\delta} \cdot \eta$

$$\delta \in \{0, 1, 2, 3, 4, 5\}, \eta \in U(-1, 1)$$

REM

$$P_{K,N}(E_0) = \int P(\varepsilon_1, \cdots, \varepsilon_K) \delta \left(E_0 - \sum_{i=1}^{K} \varepsilon_i \right) \prod_{i=1}^{K} \mathrm{d}\varepsilon_i$$

Laplace transform, simplification and approximation for $N \to \infty$, guessing of suitable scaling leads to N-independent form: F_K, with known Laplace transform