Surprising Effects of Inhomogeneity on Opinion Dynamics

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March 19, 2020

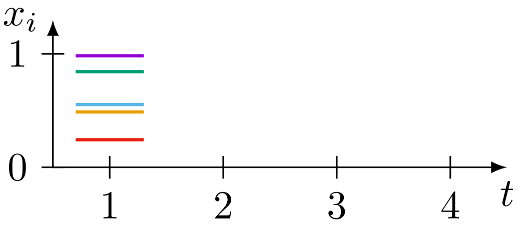
Introduction

- Opinion dynamics
  evolution of opinions in a society of agents with time
- Social influence
  agents communicate and their opinion become more similar
- Bounded confidence
  very dissimilar agents do not have influence

Can we observe complex emergent behavior?

Hegselmann-Krause bounded confidence model

- N agents
- each with opinions $x_i \in [0, 1]$
- each with confidence $\varepsilon_i \in [0, 1]$
- neighbors are similar agents j, with $|x_i - x_j| \leq \varepsilon_i$
- compromise with your neighbors $x_i(t+1) = \frac{1}{|\mathcal{N}|} \sum_{j \in \mathcal{N}} x_j(t)$
- possible stationary states: *consensus* or *fragmentation*
- measure mean size of largest cluster $\langle S \rangle$ to detect consensus



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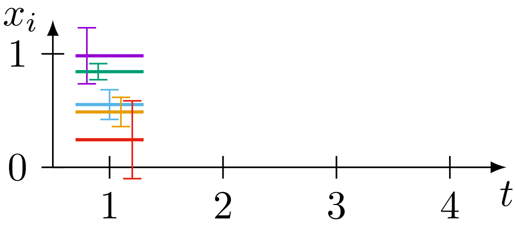

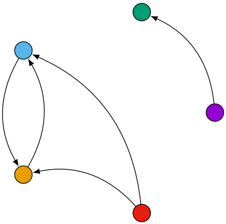

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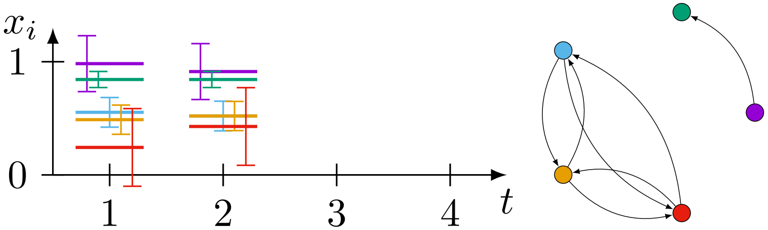

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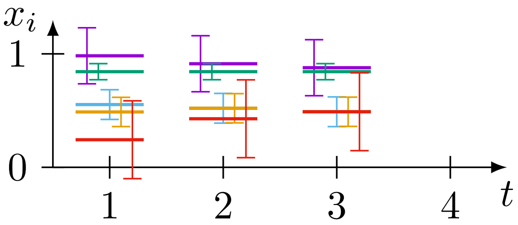

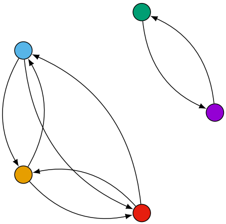

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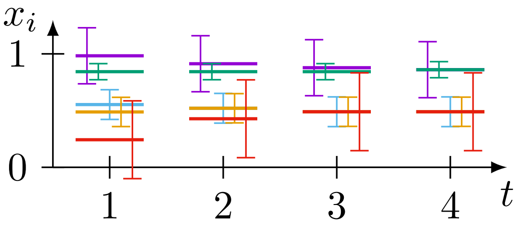

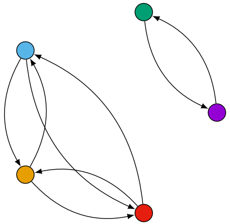

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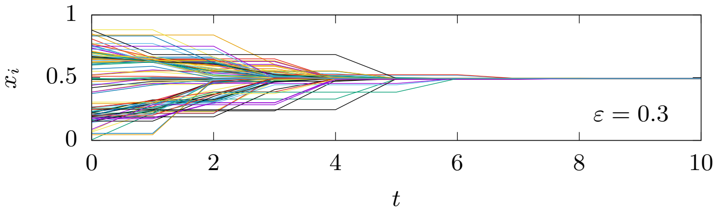

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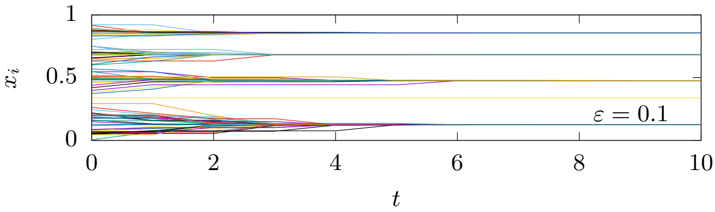


For which ε_i do we expect consensus?

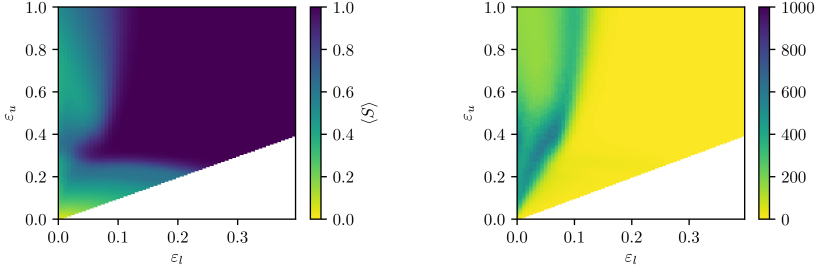
- $\varepsilon_i = \varepsilon \gtrsim 0.2$ always consensus (for large N) [1]
- larger ε_i typically lead faster to consensus

Which influence has heterogeneity in ε_i?

- bimodal $\varepsilon_i \in \{\varepsilon_1, \varepsilon_2\}$ for small systems or related models, suggest complex behavior
- Will this be stronger for stronger inhomogeneity $\varepsilon_i \in U(\varepsilon_l, \varepsilon_u)$?
- Will it be preserved for large N?

[1] Hegselmann, Krause, 2002

Exploring the Landscape



$N = 256$

- Phase space with nonmonotonous, complex structure
- Consensus where mean confidence $\varepsilon < 0.2$
- Surprising: Increasing confidence $\varepsilon_u \Rightarrow$ loss of consensus
- All effects are stronger with larger systems

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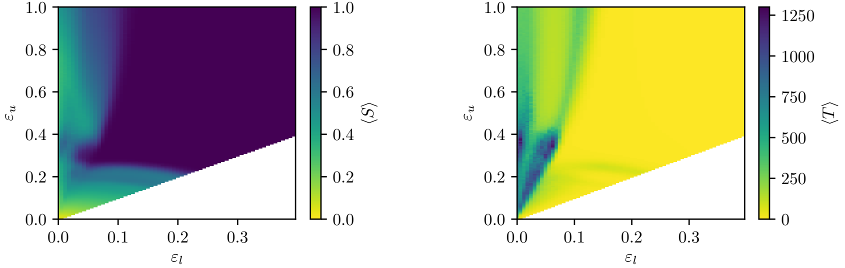

$N = 1024$

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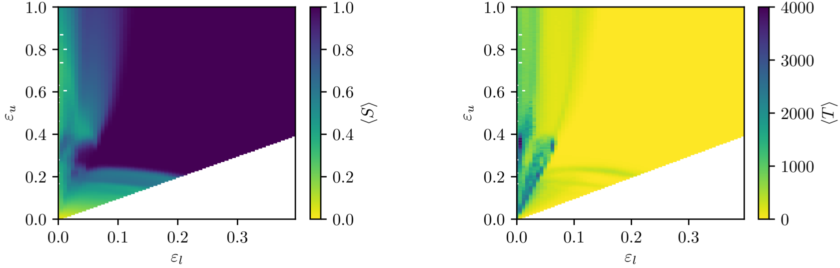

$N = 4096$

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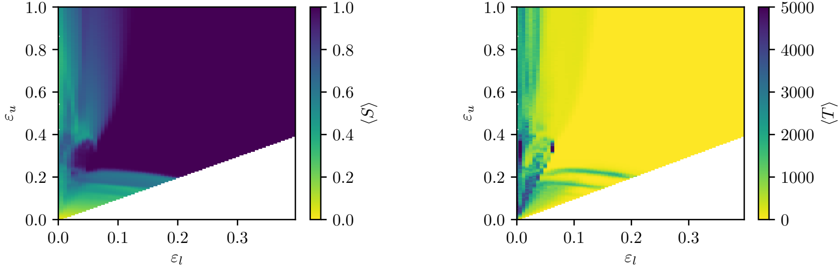

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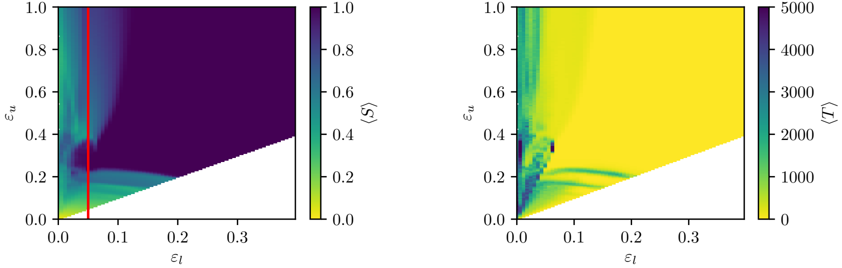

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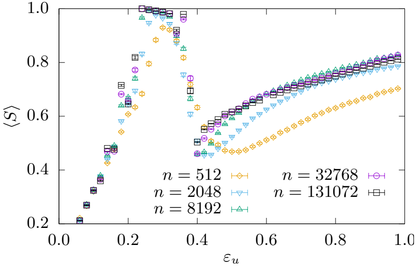
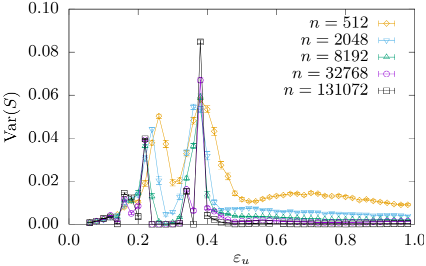
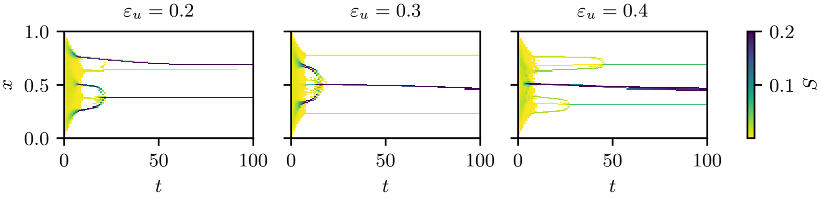
Looking closer at the reentrant phase

- $\varepsilon_i \in [0.05, \varepsilon_u]$
- increasing upper bound $\rightarrow$ more confident agents
- Two sharp flanks, getting sharper for larger systems
- Variance diverges at the flanks
- Phase transition to consensus and out of consensus





How do confident agents destroy consensus?



- local clusters develop slowly and stabilize out of range → fragmentation

- there are (almost) always 2 local clusters, which develop slowly
- attraction via moderate agents interacting with a small central cluster

- central cluster attracts confident agents very fast
- skeptic agents are left behind
- leads to fragmentation of the central cluster

Conclusions

- heterogeneity facilitates consensus
- surprising: increasing the confidence can reduce the consensus
- read more: `arxiv:2001.06877`

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Outlook So, does heterogeneity always lead to more consensus in society?

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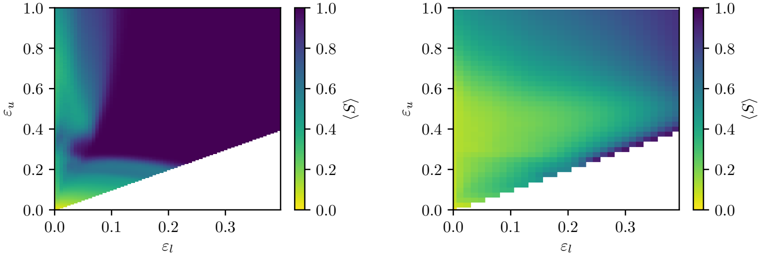
Outlook So, does heterogeneity always lead to more consensus in society?
Well, not necessarily.
introduction of a small cost:

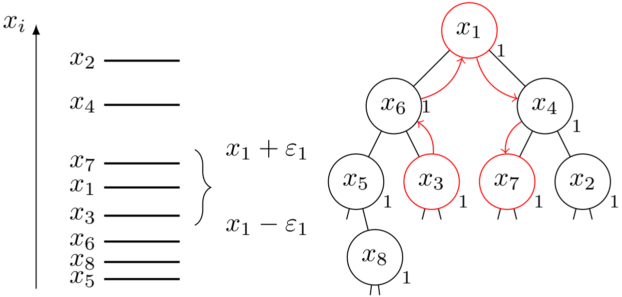


What is the problem when simulating? Introducing a faster algorithm.

- At each time step each agent has to average over all neighbors $\Rightarrow \mathcal{O}(N^2)$
- Introducing new algorithm
  - It is only necessary to touch the neighbors, which are far fewer for low ε_i
  - Converged clusters look for another agent like a single agent with high weight
- allows us to gather good statistics for systems two orders of magnitude larger ($N = 131072$) than what is typically studied

Introducing a faster algorithm.

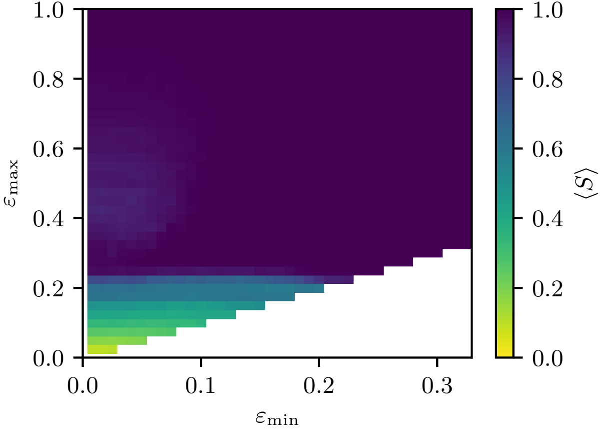
- Save all opinions in the system in a tree
- to average the neighbors of agent i
  - find the smallest opinion $x_j \geq x_i - \varepsilon_i$ in $\mathcal{O}(\log(N))$
  - traverse the tree in order and stop averaging on encountering $x_j \geq x_i + \varepsilon_i$
  - if a value x_j occurs more than once in the tree, assign it a weight



What about other distributions of ε_i?

Bounded power law

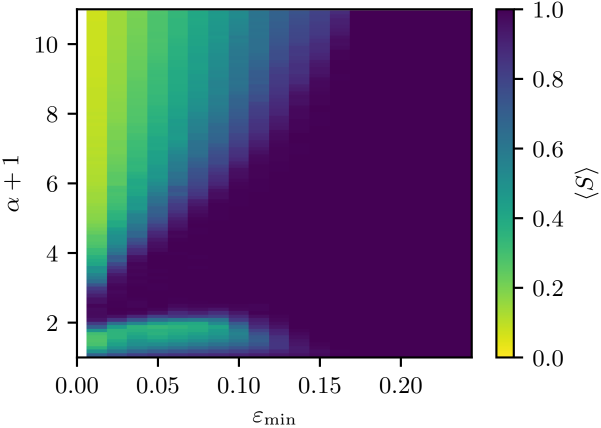
$$p(\varepsilon) = c\varepsilon^{-\gamma}$$



What about other distributions of ε_i?

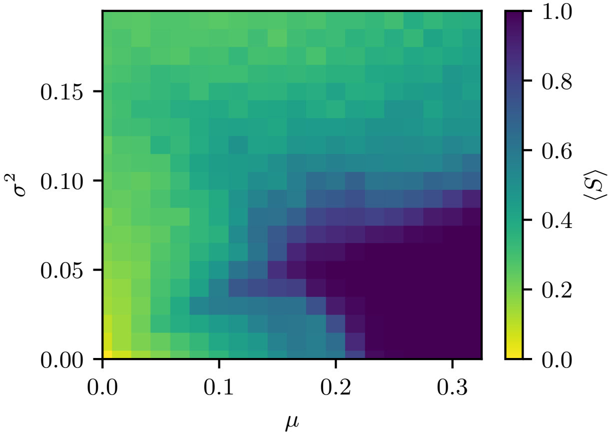
Pareto

$$p(\varepsilon) = \frac{\alpha x_{\min}^{\alpha}}{x^{\alpha+1}}$$



What about other distributions of ε_i?

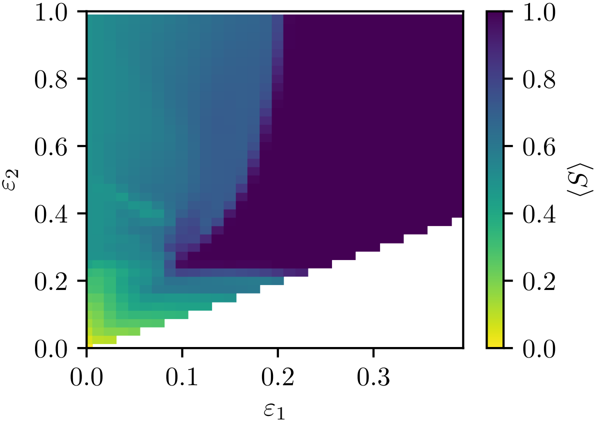
Gaussian



What about other distributions of ε_i?

Bimodal

$$p(\varepsilon) = \delta(\varepsilon - \varepsilon_1) + \delta(\varepsilon - \varepsilon_2)$$



Mean Dynamics